

# Spatial Complexity Due to Incipient Electronic Nematicity in Cuprates

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Surface probes such as scanning tunneling microscopy (STM) have detected complex patterns at the nanoscale, indicative of electronic inhomogeneity, in a variety of high temperature superconductors. In cuprates, the pattern formation is associated with the pseudogap phase, a precursor to the high temperature superconducting state. Symmetry breaking (i.e. from  $C_4$  to  $C_2$ ) in the form of electronic nematicity has recently been implicated as a unifying theme of the pseudogap phase,<sup>1</sup> however the fundamental physics governing the nanoscale pattern formation has not yet been identified. Here we use universal cluster properties extracted from STM studies of cuprate superconductors in order to identify the fundamental physics controlling the complex pattern formation. We find that the pattern formation is set by a delicate balance between disorder and interactions, leading to a fractal nature of the cluster pattern. The method introduced here may be extended to a variety of surface probes, enabling the direct measurement of the *dimension* of the phenomenon being studied, in order to determine whether the phenomenon arises from the bulk of the material, or whether it is confined to the surface.

While lanthanum-based cuprate superconductors display striking evidence of rotational symmetry breaking in the pseudogap regime, sometimes even leading to translational symmetry breaking<sup>2</sup>, such issues have been harder to settle in the higher transition temperature compounds such as YBCO and BSCCO. Recent experimental progress on YBCO has shed new light, including nematic behavior in the pseudogap regime detected via Nernst effect<sup>3</sup>, transport<sup>4</sup>, and neutron scattering<sup>5</sup>, as well as evidence of time-reversal symmetry breaking detected via neutron scattering<sup>6</sup> and the Kerr effect<sup>7</sup>. The detection of a glass of unidirectional domains via STM in NCCOC and Dy-Bi2212 has now been followed by the dramatic demonstration of rather large electron nematic domains in BSCCO<sup>1,8</sup>.

Because of the fragile nature of rotational symmetry breaking, certain classes of disorder forbid long-range orientational order in 2D systems, and significantly suppress the order in strongly layered systems.<sup>9,10</sup> It is therefore critical to understand the effects of both quenched disorder and interactions between nematic domains in these systems. As shown in Ref. 9, the orientational degree of freedom of the electron nematic in the cuprates in the presence of quenched disorder maps to a disordered Ising model in the following way. When an electron nematic forms, there is a preferred orientation to the electronic degrees of freedom. We consider Cu-O planes which are locally  $C_4$  symmetric, with an incipient electron nematic which breaks the local rotational symmetry of the host crystal from  $C_4$  to  $C_2$ , leading to two possible nematic orientations. We coarse grain the system, and define a local nematic order parameter by  $\sigma = \pm 1$ , corresponding to the two allowed orientations. The tendency for neighboring nematic regions to align is modeled as a ferromagnetic nearest-neighbor interaction.

Material disorder in the form of, *e.g.*, dopant atoms competes with the ferromagnetic coupling between local nematic directors. Upon coarse graining, there are

two broad classes of disorder which present themselves at the order parameter level: local energy density disorder (which includes random  $T_c$  disorder), and random field disorder. Local density disorder may arise in the form of, *e.g.*, random bond disorder, in which the strength of the ferromagnetic coupling varies from coarse-grained site to coarse-grained site in the system. In addition, the local amplitude of the nematic order parameter can vary spatially<sup>1</sup>. In an order parameter description, this type of disorder may be subsumed into randomness in the bond strengths. The other class of disorder, random field disorder, arises when the pattern of doping atoms breaks rotational symmetry, thus favoring one or the other orientation of the nematic director in that region. This type of disorder couples linearly to the nematic director. The purpose of this paper is to gain an understanding, at the order parameter level, of both the relative importance of interactions and disorder, and also to discern the type of disorder, thus arriving at an order parameter level description of the fundamental physics controlling the inhomogeneous pattern formation observed at the nanoscale in these materials. We consider a general model encompassing both classes of disorder:

$$H = - \sum_{\langle ij \rangle_{||}} J^{||} (1 + \delta J_{ij}^{||}) \sigma_i \sigma_j \quad (1)$$

$$- \sum_{\langle ij \rangle_{\perp}} J^{\perp} (1 + \delta J_{ij}^{\perp}) \sigma_i \sigma_j - \sum_i (h + h_i) \sigma_i .$$

Here,  $J^{||}$  sets the overall strength of the in-plane ferromagnetic coupling between nearest neighbor Ising nematic variables, is an in-plane ferromagnetic coupling between nearest neighbor Ising nematic variables, and  $\delta J_{ij}^{||}$  represents bond disorder in the coupling strength.  $J^{\perp}$  represents the overall coupling strength between Ising variables in neighboring planes, and  $\delta J_{ij}^{\perp}$  is bond disorder in the interplane coupling strength. The random field  $h_i$  is chosen from a gaussian probability distribution cen-

tered about zero, with width  $\Delta$ , which we call the “random field strength”. We have chosen to discretize the system on a cubic lattice for simplicity, a choice which does not affect the universal properties of the model as long as the mean value of the random bond couplings remains positive.

The field  $h = h_{\text{int}} + h_{\text{ext}}$  represents an orienting field which breaks rotational symmetry. The external contribution  $h_{\text{ext}}$  may be achieved by the application of, *e.g.*, magnetic fields, uniaxial pressure, or high currents among others. In the systems we have analyzed, data was taken in the absence of applied fields. Another source of finite  $h$  may be internal crystal effects  $h_{\text{int}}$ , such as the chains in YBCO may present. Such issues do not arise in NCCOC or BSCCO, and it is appropriate in both NCCOC and BSCCO to set the internal field  $h_{\text{int}} = 0$  in our model.

The order parameter  $m = 1/N \sum_i \sigma_i$  describes the degree of orientational order in the system. In the limit of zero random field strength and no bond disorder, the model has a finite-temperature continuous phase transition for dimension  $d \geq 2$ , from a disordered phase to an ordered electron nematic, *i.e.* with long range orientational order  $\langle m \rangle \neq 0$ . However, for any finite random field strength, long range orientational order is *forbidden* in two dimensions. In three dimensions, there is a finite critical disorder strength of the random field,  $\Delta_c^{3D} = 2.27J_{\parallel}$ .<sup>11</sup> In strongly layered systems, the critical disorder strength is finite, but significantly reduced from the 3D value.<sup>10</sup> In contrast with random field models, weak random bond disorder does not forbid nematic order in two dimensions; rather, the phase transition is governed by clean Ising model exponents. In layered and three dimensional systems, the phase transition of the random bond model is controlled by the disordered fixed point “R”. In the presence of both random bond and random field disorder, the universality class is that of the random field model.

## I. UNIVERSAL SPATIAL PROPERTIES OF CLUSTERS

Fig. 1 shows our extraction of locally oriented (nematic) domains from scanning tunneling microscopy data on  $\text{Bi}_2\text{Sr}_2\text{Dy}_{0.2}\text{Ca}_{0.8}\text{Cu}_2\text{O}_{8+\delta}$  (Dy-Bi2212) from Fig. S3 of Ref.<sup>24</sup>, reported as an “R-map,” where at each position  $\vec{r}$ ,  $R(\vec{r}, V) = I(\vec{r}, +V)/I(\vec{r}, -V)$  is the ratio of the tunneling current at positive voltage to that at negative voltage.<sup>24–26</sup> Based on the “R”-map, the presence of local, unidirectional domains of width  $4a_o$  were noted<sup>24</sup>, corresponding to the distance between “legs” of the  $4a_o$ -wide ladders, where  $a_o$  is the Cu-Cu distance within the Cu-O planes. There is also a coexisting local density wave at  $a_o$ , corresponding to the distance between “rungs” on the ladders. To effect the mapping to Ising nematic domains, we take a local spatial Fourier transform (FT) of the R-map in any given region, and then focus on the  $a_o$  local density wave. There are two “flavors” to this

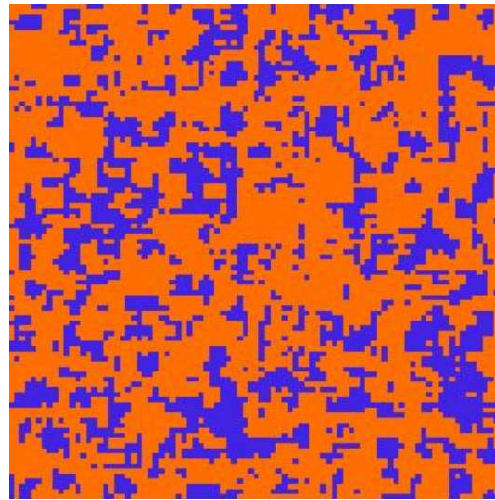


FIG. 1. Extraction of Ising domains from tunneling asymmetry maps in Dy-Bi2212, with  $T_c \approx 45\text{K}$ , taken at  $T = 4.2\text{K}$ . Notice the large, percolating cluster (orange), with isolated flipped domains inside.

density wave, either oriented along the  $a$  axis, or along the  $b$  axis, depending on the local orientation of the ladders. The relative weight of the two density waves can be discerned from comparing the  $a_o$  periodicity peaks in the local FT, comparing the weight of the  $a_o$  peak in the  $a$  direction to that in the  $b$  direction. We assign a local Ising variable based on this relative weight. We have checked that our results are insensitive to details such as the size of the FT window and the Ising lattice spacing.<sup>27</sup> Note that our assignment of the Ising cluster map is independent of the local strength of the nematicity, which can be subsumed into randomness in the Ising couplings  $J_{ij}$ , contributing to random bond disorder.

The extracted cluster maps resemble cluster patterns observed in numerical studies of Eqn. 2,<sup>11</sup> consistent with the idea of mapping disordered nematics to a disordered Ising model.<sup>9</sup> We show in Tables I and II cluster properties which can be extracted from the spatial configurations of clusters, as defined in the following references:  $\eta$  is the spin-spin correlation function exponent<sup>17</sup>,  $\tau$  is the cluster size distribution exponent<sup>29</sup>, and  $d_c$  is the fractal dimension of the spanning cluster<sup>11</sup>. The theoretical values available from the literature are shown for clean Ising models, random bond Ising models, and random field Ising models in both two and three dimensions,<sup>14,16,30</sup> or derived from a combination of theoretical exponents available in the literature and scaling relations. (See the Appendix.) Notice how *distinct* the numbers are from one universality class to the next. This implies that different universality classes are sufficiently different that one can in principle use these methods to identify the universality class controlling the inhomogeneous pattern formation. In particular, because critical exponents are so sensitive to dimension, this method can in principle be used to allow any surface probe to *directly measure*

	Clean Ising Model	Random Bond Disorder	Random Field Disorder
$\tau$	2.067 <sup>12,13</sup>	2.067 <sup>12,13</sup>	1.6 <sup>14</sup>
$d_c$	1.125 <sup>11,15</sup>	1.125 <sup>11,15</sup>	1.9 <sup>16</sup>
$d - 2 + \eta$	0.25 <sup>15</sup>	0.25 <sup>15</sup>	1 <sup>17</sup>

TABLE I. **Theoretical Critical Cluster Exponents of Two Dimensional Ising Models.** In two dimensions, random bond disorder is irrelevant, and the order to disorder phase transition is controlled by the clean Ising model critical point. For the random field Ising model in two dimensions, long range order is forbidden at finite disorder strength. However, there is an unstable fixed point at  $T=0$  and zero disorder. The table reports values from the literature for this point, since it may affect the scaling in some regimes.

	Clean Ising Model	Random Bond Disorder	Random Field Disorder
$\tau$	2.21 <sup>12,13</sup>	2.221 <sup>12,18</sup>	2.02 $\pm$ 0.03 <sup>19,20</sup>
$d_c$	2.11 <sup>11,15</sup>	1.98 <sup>11,18</sup>	2.78 $\pm$ 0.05 <sup>20-22</sup>
$d - 2 + \eta$	1.04 <sup>15</sup>	1.06 <sup>18</sup>	1.5 $\pm$ 0.05 <sup>23</sup>

TABLE II. **Theoretical Critical Cluster Exponents of Three Dimensional Ising Models.**

	Dy-Bi2212 at T=4.2K
$\tau$	1.71 $\pm$ 0.07
$d_c$	1.56 $\pm$ 0.02
$d - 2 + \eta$	0.26 $\pm$ 0.03

TABLE III. **Critical Cluster Exponents from STM on the cuprate superconductor Dy-Bi2212.**

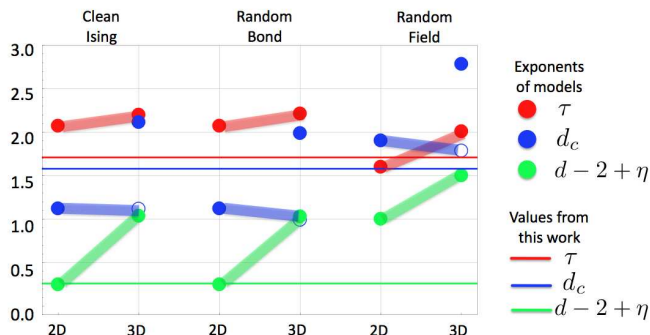


FIG. 2. Chart comparison of our results for Dy-Bi2212 compared to the theoretical models. Solid circles represent the values reported in the literature, summarized in Tables I and II. Red circles represent  $\tau$ , blue circles represent  $d_c$ , and green circles represent  $d - 2 + \eta$ . In comparing our results to the 3D models, we have assumed that at the surface, one observes a two-dimensional cross section of a 3D cluster, and we subtract 1 from the fractal dimension  $d_c$ . These values are represented by the open blue circles. Thick lines connecting circles represent putative crossover of exponents from 2D to 3D behavior in a layered material. The horizontal lines in red, blue, and green are our results for the spatial characteristics of clusters in Dy-Bi2212 as reported in Fig. S3 of Ref. 28

the dimension of the phenomenon studied, to determine whether the phenomenon is happening only on the surface of the material ( $d = 2$ ), or whether it arises from the bulk of the material ( $d = 3$ ).

We apply quantitative cluster analysis

methods<sup>11,20-22,29</sup> from the statistical mechanics of disordered systems to identify the fundamental physics controlling the inhomogeneous pattern formation. Our results using published STM data<sup>24</sup> on Dy-Bi2212, a material which displays evidence of a disordered electron nematic at the surface,<sup>1,24</sup> are shown in Table III. Fig. 2 charts a direct comparison between our values extracted from data, and the theoretical fixed point exponents of disordered Ising models. Because of the strongly layered nature of Dy-Bi2212, we expect there to be a regime of scaling which follows the 2D universality class, which then crosses over to the 3D universality class at longer length scales.<sup>10</sup>

### A. Cluster Size Distribution

The exponent  $\tau$  characterizes the distribution of cluster sizes. Fig. 1 shows the clusters identified from Dy-Bi2212. Here, a single cluster is a nearest-neighbor set of like-oriented Ising variables on the square lattice we have chosen to use. Fig. 3 shows the cluster size distribution, a histogram of cluster sizes. The results are noisier at large cluster size, due to the finite field of view (FOV). Fig. 4 shows the same distribution with logarithmic binning. Near a critical point, the cluster size distribution is of the form  $D(A) \propto A^{-\tau}$ . Using a straightforward fit to this form, we find  $\tau = 1.71 \pm 0.07$ . There is one large spanning cluster, represented by the last point in Fig. 4. Near criticality, scaling can be observed over a finite range. Note that in this case, the spanning cluster is also in the regime of scaling, and almost 4 decades of scaling are present in this cluster property, and this extraction of the exponent  $\tau$  can be considered quite reliable.

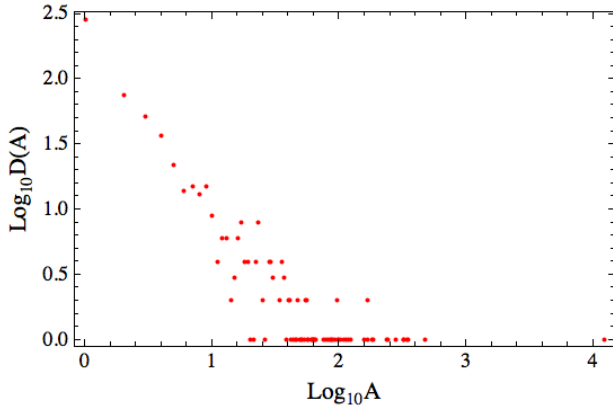
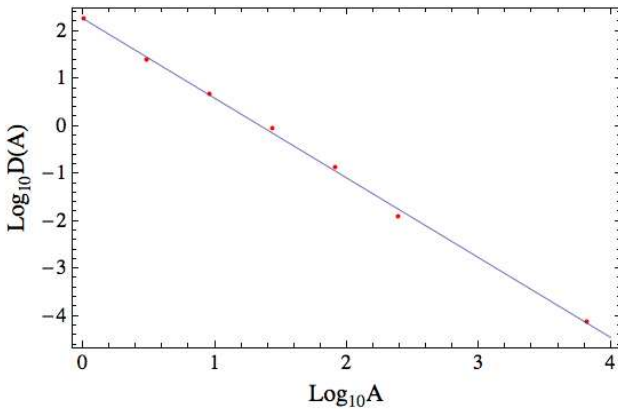


FIG. 3. Raw cluster size distribution.

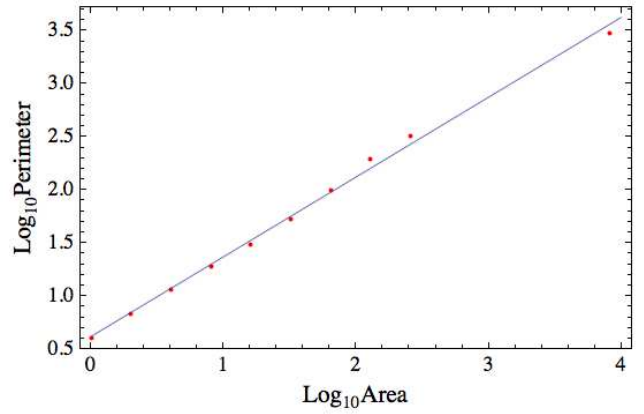
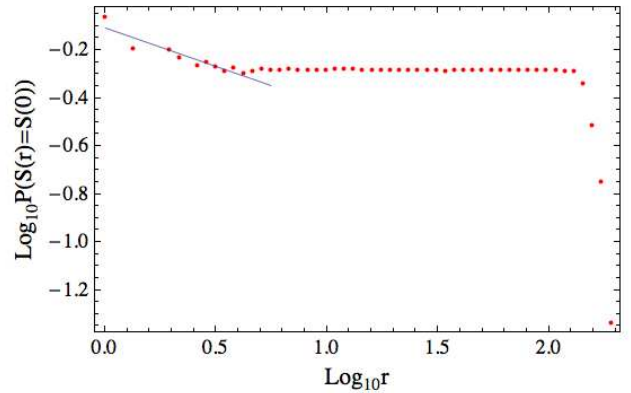
FIG. 4. Cluster Size Distribution after logarithmic binning, used to calculate the critical exponent  $\tau$ .

### B. Fractal Dimension of Clusters

The fractal dimension of clusters relates the surface area of each cluster to its volume. Because STM is a surface probe, the available information is two-dimensional, thus the fractal dimension of clusters relates their perimeter  $p$  to the area  $a$  of each cluster:  $p \propto a^{d_c/d}$ .<sup>11</sup> Using  $d = 2$  because only a two dimensional cross section of the cluster properties is available, a straightforward fit gives  $d_c = 1.56 \pm 0.02$ . It is evident from Fig. 5 that the spanning cluster is also in the scaling regime for this measure, leading to almost 4 decades of scaling for the fractal dimension, and as with  $\tau$ , this extraction of the exponent  $d_c$  can also be considered quite reliable.

### C. Spatial Correlations

Within the Ising description, the spin-spin correlation function of the Ising pseudospin variables is described by  $G(r) \propto |r|^{-(d-2+\eta)}$ . Within the field of view available, we have averaged over all sites to obtain the spin-spin

FIG. 5. The fractal dimension of clusters,  $d_c$ , relates the perimeter of clusters to their area. (See text for details.) Logarithmic binning has been used.FIG. 6. Logarithmically binned data for calculating the critical exponent  $\eta$ .

correlation function plotted in Fig. 6. At short distances, a weak region of scaling is evident, which extends roughly half of a decade. With only a half decade of scaling, this measure should be considered *unreliable*, especially since the presence of an as-yet undetermined scaling function may skew results at short distances. Nevertheless, for completeness, we note that the value of  $d-2+\eta$  obtained from this simple fitting procedure is  $d-2+\eta = 0.26 \pm 0.03$ .

## II. DISCUSSION

The value of the cluster size distribution exponent  $\tau$  displayed by the clusters in Dy-Bi2212 lies between the 2D and 3D values of the RFIM, indicating that the cluster size distribution is consistent with the scaling of a *layered* RFIM. By contrast, the value of  $\tau$  is 17% away from the clean and random bond values denoted by thick red lines in Fig. 2. Furthermore, the data show almost 4 decades of scaling for  $\tau$  indicating the value of  $\tau$  extracted is highly reliable. In comparing the fractal dimension of clusters  $d_c$



to 3D models, we make the ansatz that we have analyzed a 2D cross section of a 3D cluster, and subtract 1 from the expected value of  $d_c$ . We see then that the value of  $d_c$  extracted from experiment is close to the theoretical RFIM values (within 12%), whereas it is 39% away from the random bond and clean models. There are almost 4 decades of scaling for the fractal dimension  $d_c$ , making this a reliable extraction of this universal cluster property. The value of  $d - 2 + \eta$  extracted from a simple fit to the spin-spin correlation function is inconsistent with the random field universality classes, but it is consistent with Ising models, either clean or random bond models close to two dimensions. However, this is the least reliable exponent extracted, as it displays less than one decade of scaling. The most reliable properties we have extracted are  $\tau$  and  $d_c$  with almost 4 decades of scaling including the spanning cluster. (We note that even without the spanning cluster, over 2 decades of scaling are present in both of these quantities.) Overall, there is a striking similarity between the spatial properties of clusters observed in STM and universal cluster properties of disordered Ising models in 2D and 3D, particularly of the random field type.

The presence of power law scaling in these quantities suggests that the pattern formation in Dy-Bi2212 is near criticality. Because the critical region is large in the RFIM it is possible to observe the scaling in a broad range of parameters near the critical point.<sup>31</sup> For example, two decades of scaling appear in the avalanche size distribution of the nonequilibrium 3D RFIM even at a disorder strength of  $\Delta = 4J$ , which is 85% larger than the nonequilibrium critical disorder strength  $\Delta_c^{3D} = 2.16J$ .<sup>31</sup> The presence of power law scaling is also indicative of the fractal nature of this pattern formation, suggesting a potential connection to recent work on the fractal nature of doping in some high temperature superconductors.<sup>32</sup>

In a true 2D condensed matter system (*e.g.*, with a finite disorder concentration), random field disorder would forbid long-range order of an electron nematic. However, in any finite size 2D system, the system can *appear* to be ordered whenever the correlation length  $\xi$  is larger than the system size observed. Although the material we have considered is strongly layered, the layered RFIM ultimately flows to the 3D universality class,<sup>10</sup> which has a finite critical disorder strength. Because of the weak coupling between planes, the critical disorder strength is suppressed from the 3D limit, *i.e.*  $R_c(J_\perp < J_\parallel < R_c^{3D})$ . Based on this, coupled with the fact that long-range-ordered nematicity has not been experimentally detected in BSCCO via a bulk probe, it is reasonable to expect the disorder strength in this material to be beyond the critical disorder strength. However, there is a spanning cluster in Fig. 1, so the system cannot be too far above the critical disorder strength, since deep in the disordered phase, clusters are small. A spanning cluster was also observed in Ref. 1. We thus conclude that this material is in the *intermediate disorder regime*,<sup>10</sup> where the disorder strength is small within a plane, but strong between

planes,  $J_\parallel \gg \Delta \gg J_\perp$ . Although long-range nematic order is not possible in this regime, clusters within each plane can have a long correlation length.

### III. CONCLUSIONS

In conclusion, we have used quantitative cluster methods from the statistical mechanics of disordered systems to identify the fundamental physics controlling the complex pattern formation revealed by STM in Dy-Bi2212. We find that the patterns are scale-invariant and fractal, indicating that Dy-Bi2212 is near a critical point, due to the interplay of local rotational symmetry breaking (electronic nematicity) with quenched disorder. We furthermore identify this critical point as being in the Ising universality class, with random field disorder the dominant type of disorder. We find that the disorder strength is in the *intermediate* regime, with disorder being a weak effect within each plane (leading to large clusters within a plane), while disorder is a strong effect in the interplane direction (leading to a lack of true long range order), indicating the pattern formation may best be described by a layered random field Ising model. Furthermore, this method may be extended to other materials and surface probes. For spatial patterns that may be tuned through or sufficiently near a critical point, this allows the determination of the dimension of the phenomenon being studied, enabling surface probes to distinguish whether the pattern formation is merely on the surface, or extends throughout the bulk of the material.

### IV. ACKNOWLEDGMENTS

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### APPENDIX

#### A. Theoretical Values of Critical Exponents

The exact critical exponents we require in order to compare to known theoretical results of Ising models are not necessarily directly in the literature. However, scaling relations allow us to infer values for the cluster size distribution exponent,  $\tau$ , and the fractal dimension of clusters,  $d_c$ . To infer  $d_c$ , we employ the scaling relation<sup>11</sup>:

$$d_c = \frac{\alpha - 1}{\nu} + d + \frac{\beta}{\nu} \quad (2)$$

To infer  $\tau$  we used the scaling relation<sup>12</sup>

$$\tau = \frac{2 - \alpha}{\beta + \gamma} + 1 \quad (3)$$

along with the Rushbrooke identity<sup>12</sup> to eliminate  $\gamma$  in favor of  $\alpha$ :

$$\alpha + 2\beta + \gamma = 2 \rightarrow \gamma = 2 - 2\beta - \alpha \quad (4)$$

to get the final expression:

$$\tau = \frac{2 - \alpha}{2 - \beta - \alpha} + 1 \quad (5)$$

because  $\beta$  is so small this equation is relatively insensitive to  $\alpha$

Some of these exponents were extracted directly from the literature, in order to avoid confusion an asterisk will be used to denote that a particular value was inferred via relations rather than directly reported in the literature.

Exponent	2D Clean Ising Model	3D Clean Ising Model
$d - 2 + \eta$	$1/4^{33}$	$1.04^{15}$
$\alpha$	$0^{33}$	$0.1118(6)^{15}$
$\beta$	$1/8^{33}$	$0.326(4)^{15}$
$\nu$	$1^{33}$	$0.6294(2)^{15}$
$\tau$	$31/15^*$	$2.21^*$
$d_c$	$9/8^*$	$2.11^*$

TABLE IV. Theoretical values of critical exponents in 2D and 3D clean Ising models.

Exponent	3D RBIM “R” point
$d - 2 + \eta$	$1.029^{34}$
$\alpha$	$-0.04^{34}$
$\beta$	$0.35^{34}$
$\gamma$	$1.34^{34}$
$\nu$	$0.68^{34}$
$\tau$	$2.21^*$
$d_c$	$1.99^*$

TABLE V. Theoretical values of critical exponents in 3D Ising models with weak random bond disorder. Note that transition in the 2D random bond Ising model is controlled by the clean fixed point.

Exponent	2D RFIM	3D RFIM
$d - 2 + \eta$	$1^{17}$	$0.5 \pm 1.03^{35}$
$\alpha$		$0.63^{35}$
$\beta$		$0.017(5)^{11}$
$\nu$		$1.37(9)^{11}$
$\tau$	$1.6^{14}$	$2.01^*$
$d_c$	$1.9^{16}$	$2.74^*$

TABLE VI. Theoretical values of critical exponents in 2D and 3D random field Ising models.

## B. Harris criterion considerations

The Harris criterion states that local energy density disorder (such as random  $T_c$  disorder and random bond disorder) is irrelevant if  $d\nu > 2$ , where the exponents refer to the clean model. In the presence of hyperscaling (obeyed by the clean and random bond cases),  $d\nu = 2 - \alpha$  implies that the Harris criterion reduces to  $\alpha < 0$ . In the 3D clean Ising model,  $\alpha \approx 0.1^{12}$ , and randomness is relevant. In the 2D clean Ising model,  $\alpha = 0$  and such randomness is marginal. For the 3D case with weak bond disorder, there is a disordered fixed point with new exponents. In the 2D case with weak bond disorder, it has been shown that the system flows toward the clean model.<sup>30,36</sup>

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